Distributed Multi-Robot Coordination with Model Predictive Control and Duality Theory

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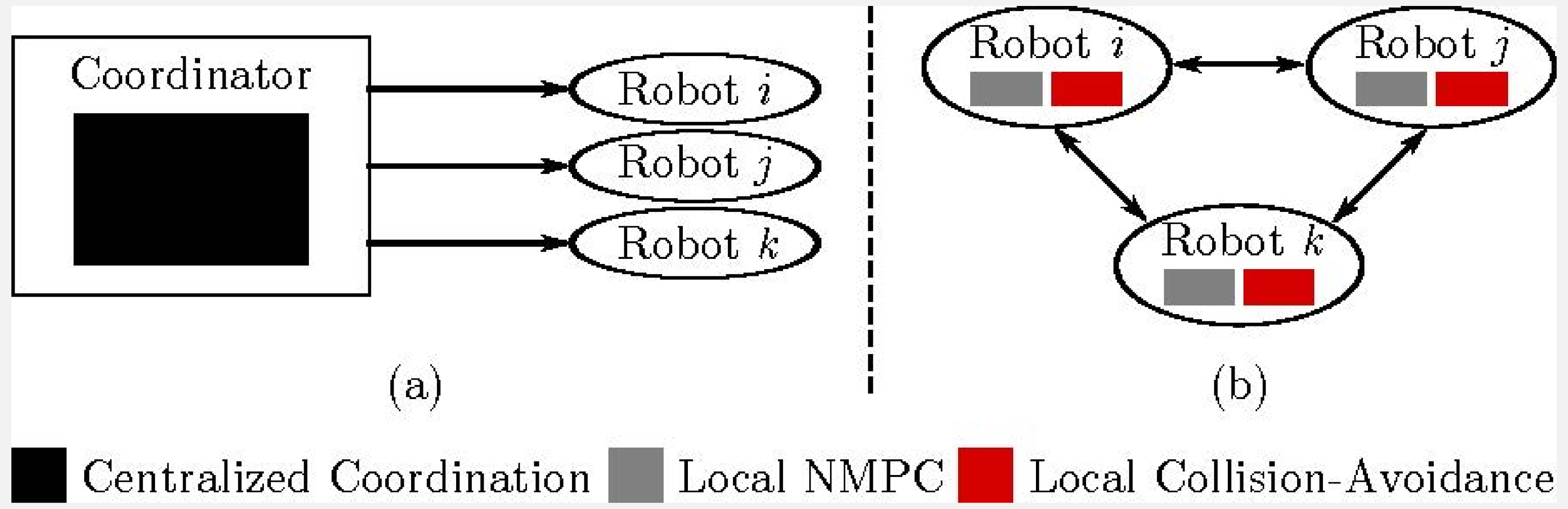
1. Introduction

Goal: safe motion planning in tight environments in a distributed way in real time.



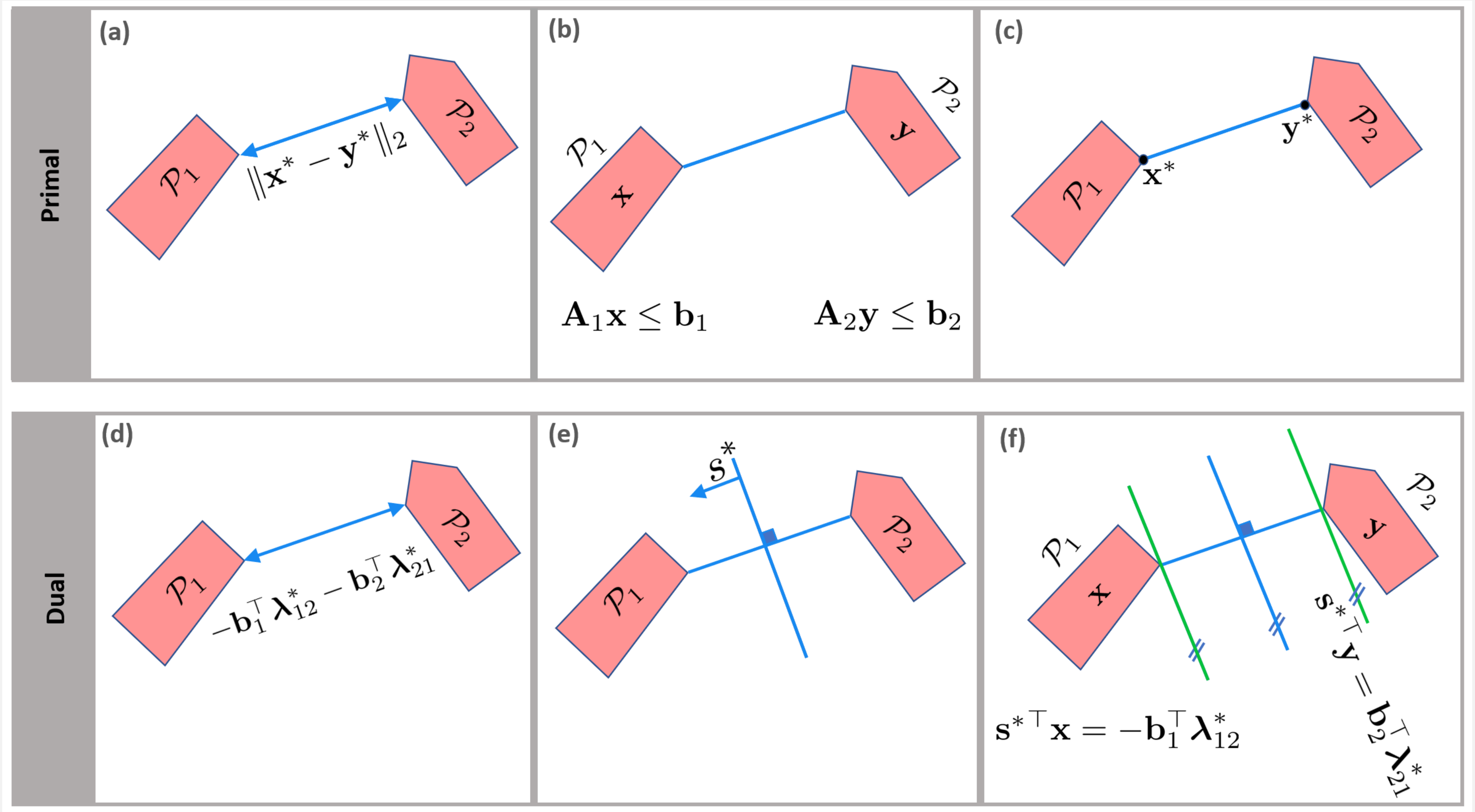
2. Collision Avoidance: Distance between Two Polytopes

Primal Formulation

$$\mathrm{dist}(\mathcal{P}_1, \mathcal{P}_2) = \min_{\mathbf{x},\mathbf{y}} \{\|\mathbf{x}-\mathbf{y}\|_2 \,|\, \mathbf{A}_1\mathbf{x} \le \mathbf{b}_1, \mathbf{A}_2\mathbf{y} \le \mathbf{b}_2\}$$

Dual Formulation

$$\mathrm{dist}(\mathcal{P}_1, \mathcal{P}_2) = \max_{\boldsymbol{\lambda}_{12},\boldsymbol{\lambda}_{21},\mathbf{s}} \; -\mathbf{b}_1^\top\boldsymbol{\lambda}_{12} - \mathbf{b}_2^\top\boldsymbol{\lambda}_{21}$$
$$\text{s.t. } \mathbf{A}_1^\top\boldsymbol{\lambda}_{12} + \mathbf{s} = 0, \; \mathbf{A}_2^\top\boldsymbol{\lambda}_{21} - \mathbf{s} = 0,$$
$$\|\mathbf{s}\|_2 \le 1, -\boldsymbol{\lambda}_{12} \le 0, \; -\boldsymbol{\lambda}_{21} \le 0$$



3. Centralized Formulation: Nonlinear MPC

$$\min_{\mathbf{u}^i(\cdot|t)} \quad \text{Cost} \; \sum_{i=1}^{M} J^i(\mathbf{z}^i, \mathbf{u}^i)$$

(State, Input)

subject to
- Dynamics: $\mathbf{z}^i(k+1|t) = f(\mathbf{z}^i(k|t), \mathbf{u}^i(k|t)),$
- Initial Condition: $\mathbf{z}^i(0|t) = \mathbf{z}^i(t),$
- State and Input Limits: $\mathbf{z}^i(k|t) \in \mathcal{Z}, \; \mathbf{u}^i(k|t) \in \mathcal{U},$
- Collision Avoidance: $\mathcal{P}(\mathbf{z}^i(k|t)) \cap \mathcal{P}(\mathbf{z}^j(k|t)) = \emptyset, \quad i \neq j$

$\forall i \in \mathcal{V}, \; j \in \mathcal{N}_i,$ and $k \in \{1, 2, .., N\}.$

Robot Index

4. Distributed Formulation: Parallelization

Splitting the centralized optimization into smaller subproblems NMPC+CA to be solved in parallel.

NMPC Optimization

$$\min_{\mathbf{u}^i(\cdot|t)} \quad J^i(\mathbf{z}^i, \mathbf{u}^i)$$

subject to
$$\mathbf{z}^i(k+1|t) = f(\mathbf{z}^i(k|t), \mathbf{u}^i(k|t)),$$
$$\mathbf{z}^i(0|t) = \mathbf{z}^i(t),$$
$$\mathbf{z}^i(k|t) \in \mathcal{Z}, \; \mathbf{u}^i(k|t) \in \mathcal{U},$$
$$\big(-\mathbf{b}^i(\mathbf{z}^i(k|t))^\top \bar{\boldsymbol{\lambda}}_{ij}(k|t)$$
$$-\bar{\mathbf{b}}^j(\bar{\mathbf{z}}^j(k|t))^\top \bar{\boldsymbol{\lambda}}_{ji}(k|t)\big) \ge d_{\min},$$
$$\mathbf{A}^i(\mathbf{z}^i(k|t))^\top \bar{\boldsymbol{\lambda}}_{ij}(k|t) + \bar{\mathbf{s}}_{ij}(k|t) = 0,$$
for all $k \in \{1, 2, .., N\},$

The bar symbol denotes fixed values.

Alternating

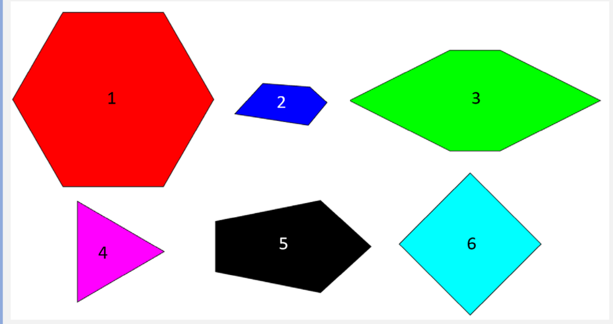
Collision Avoidance (CA) Optimization

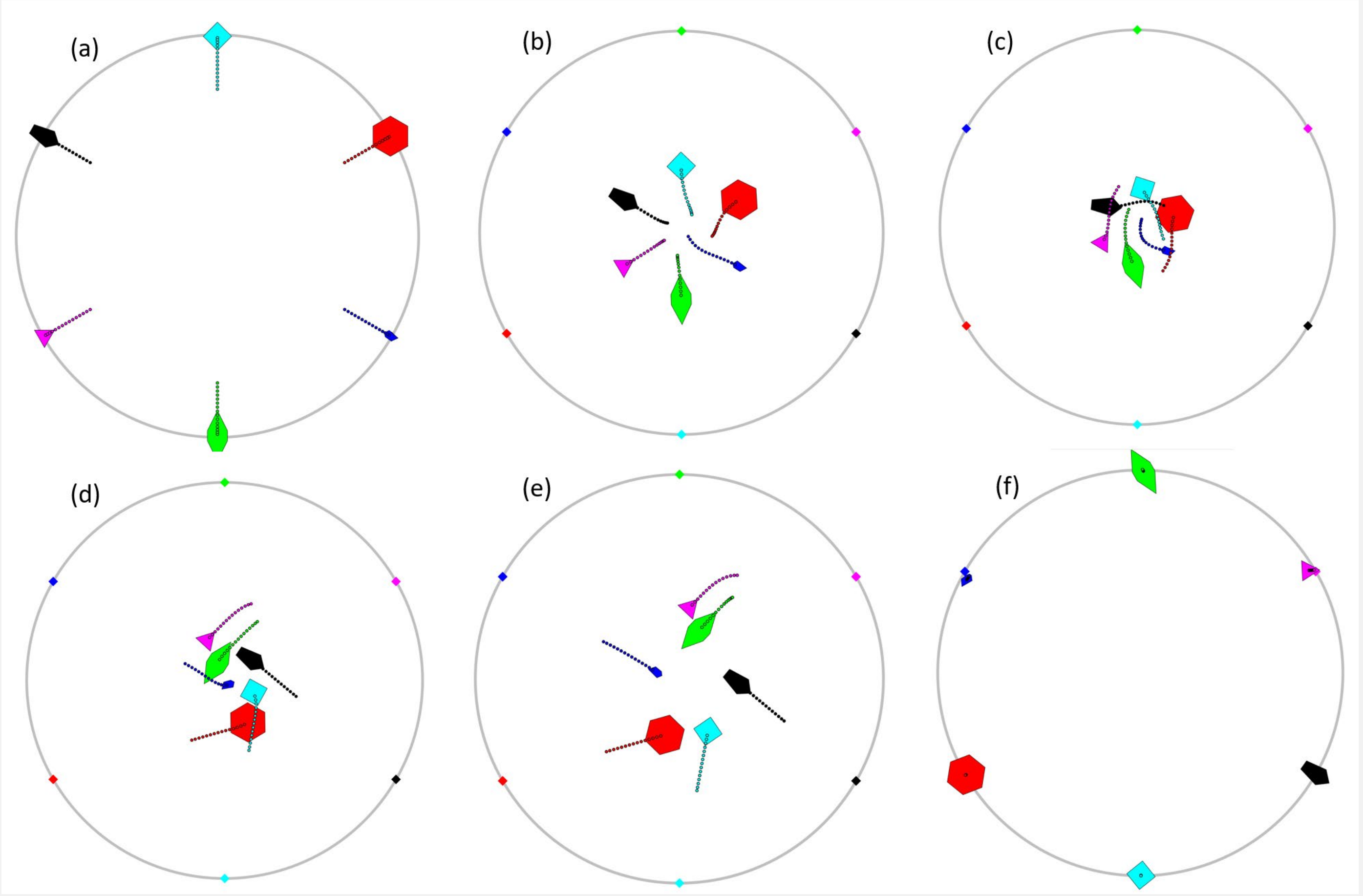
$$\max_{\substack{\boldsymbol{\lambda}_{ij}(\cdot|t),\\ \boldsymbol{\lambda}_{ji}(\cdot|t),\\ \mathbf{s}_{ij}(\cdot|t)}} \quad -\bar{\mathbf{b}}^i(\bar{\mathbf{z}}^i(k|t))^\top \boldsymbol{\lambda}_{ij}(k|t) - \bar{\mathbf{b}}^j(\bar{\mathbf{z}}^j(k|t))^\top \boldsymbol{\lambda}_{ji}(k|t)$$

subject to
$$\bar{\mathbf{A}}^i(\bar{\mathbf{z}}^i(k|t))^\top \boldsymbol{\lambda}_{ij}(k|t) + \mathbf{s}_{ij}(k|t) = 0,$$
$$\bar{\mathbf{A}}^j(\bar{\mathbf{z}}^j(k|t))^\top \boldsymbol{\lambda}_{ji}(k|t) - \mathbf{s}_{ij}(k|t) = 0$$
$$\big(-\bar{\mathbf{b}}^i(\bar{\mathbf{z}}^i(k|t))^\top \boldsymbol{\lambda}_{ij}(k|t)$$
$$-\bar{\mathbf{b}}^j(\bar{\mathbf{z}}^j(k|t))^\top \boldsymbol{\lambda}_{ji}(k|t)\big) \ge d_{\min},$$
$$\|\mathbf{s}_{ij}(k|t)\|_2 \le 1, -\boldsymbol{\lambda}_{ij}(k|t) \le 0,$$
$$-\boldsymbol{\lambda}_{ji}(k|t) \le 0, \text{ for all } i \in \mathcal{V}, \; j \in \mathcal{N}_i,$$
for all $k \in \{1, 2, .., N\}.$

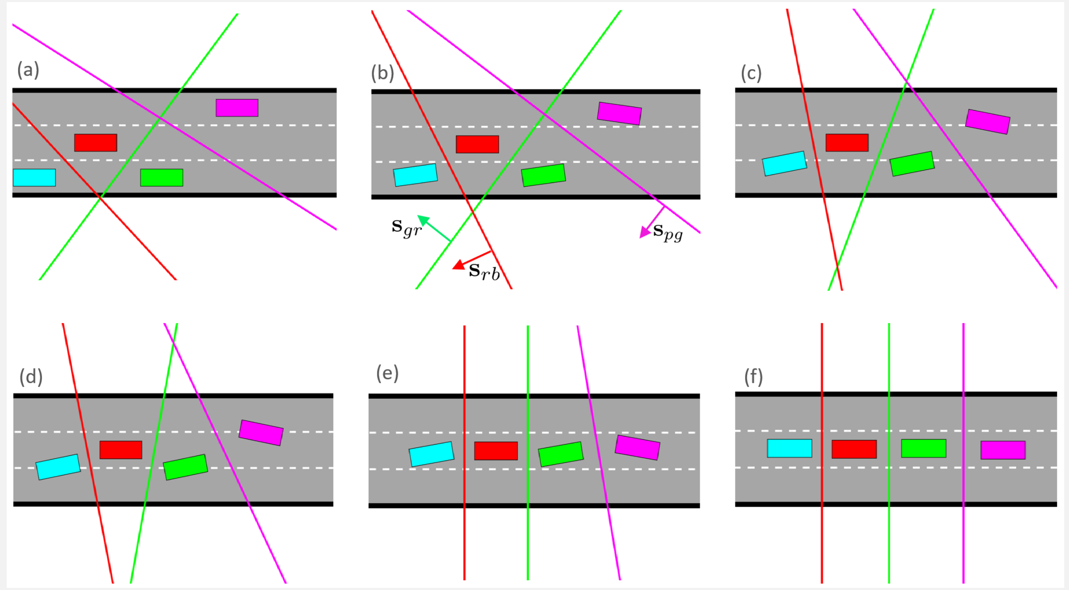
5. Results: Formation and Reconfiguration

Robots with Different Polytopic Shapes





Connected and Automated Vehicles



#	Centralized Avg.	Centralized Max.	Distributed NMPC Avg.	Distributed NMPC Max.	Distributed CA Avg.	Distributed CA Max.
2	1.3851	2.7482	0.1457	0.3621	0.0022	[illegible]
			0.1102	0.3313	0.0021	0.0022
			Total Avg. = [illegible]			
3	2.7680	4.6817	0.1848	0.3933	0.0025	0.0028
			0.1206	0.3169	0.0024	0.0026
			0.1416	0.3470	0.0022	0.0023
			Total Avg. = 0.1514			
4	12.8331	28.7763	0.1919	0.4211	0.0030	0.0024
			0.1125	0.2602	0.0027	0.0029
			0.1842	0.3497	0.0024	0.0025
			0.2041	0.4195	0.0025	0.0023
			Total Avg. = 0.1757			

Computation Time (Sec)

6. Conclusion

Using the proposed distributed algorithm the heterogenous team of robots can plan dynamically feasible trajectories in real time and navigate safely in tight environments to reach their goals.